



# Model predictive control for wastewater treatment process with feedforward compensation

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## ABSTRACT

Being an optimizing technology, model predictive control (MPC) can now be found in a wide variety of application fields. The main and most obvious control goal to be achieved in a wastewater treatment plant is to fulfill the effluent quality standards, while minimizing the operational costs. In order to maintain the effluent quality within regulation-specified limits, the MPC strategy has been applied to the Benchmark Simulation Model 1 (BSM1) simulation benchmark of wastewater treatment process. After the discussion of open loop responses of outputs to manipulated inputs and measured influent disturbances, the strategies of feedback by linear dynamic matrix control (DMC), quadratic dynamic matrix control (QDMC) and nonlinear model predictive control (NLMPC), and improvement by feedforward based on influent flow rate or ammonium concentration have been investigated. The simulation results indicate that good performance was achieved under steady influent characteristics, especially concerning the nitrogen-related species. Compared to DMC and QDMC, NLMPC with penalty function brings little improvement. Two measured disturbances have been used for feedforward control, the influent flow rate and ammonium concentration. It is shown that the performance of feedforward with respect to the influent ammonium concentration is much higher than for the feedforward with respect to the influent flow rate. However, this latter is slightly better than the DMC feedback. The best performance is obtained by combining both feedforward controllers with respect to the influent ammonium concentration and flow rate. In all cases, the improvement of performance is correlated with more aeration energy consumption.

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## 1. Introduction

The control of wastewater treatment plants is difficult because of frequent and important changes of load in flow rate and quality and also because of the biological processes which are fundamentals of the plant operation. Many control strategies have been proposed in the literature for wastewater treatment plants [1–4], but their evaluations and comparisons are difficult. This is partly due to the variability of the influent, to the complexity of the physical and biochemical phenomena and to the large range of time constants (from a few minutes to several days) which are inherent in the activated sludge process. Furthermore, the proposed control strategies differ with respect to their objectives and methods. Sometimes, the objectives are limited like dissolved oxygen control [3,5] and nitrate control [6–12], sometimes they are very large extending beyond the wastewater treatment plant like the integrated hierarchical predictive control of a wastewater treatment plant together with the sewer system [2]. The control methods include simple control [13],

feedforward–feedback control [14], linearized and optimal control [15,16], nonlinear control [17,4], fuzzy control [18], optimal control [19], supervisory control [20], model predictive control [1,5,2,11]. It should be noted that few experimental validations have been performed [20] for many reasons including lack of adequate hard or soft sensors, actuators and process control systems. However, the need for better instrumentation, control and automation is recognized. In addition, the controlling results lack standard criteria to be evaluated. A benchmark, i.e. a simulation environment defining a plant layout, a simulation model including influent loads, test procedures and evaluation criteria has been proposed within the framework of COST Actions 682 and 624 [21–23].

MPC is widely used and accepted in the industry in general and process industries in particular. A general objective of MPC schemes is to maintain the controlled variables close to their set points while respecting process operating constraints. MPC was first introduced by [24] as model algorithmic control (MAC) where the accent was set on the key role of digital computation and modelling. [25] gives a good overview of both linear and nonlinear commercially available MPC technologies. More and more severe regulations are imposed to wastewater treatment plants that are inherently multivariable processes. Clearly, an optimizing control is desired and constraint

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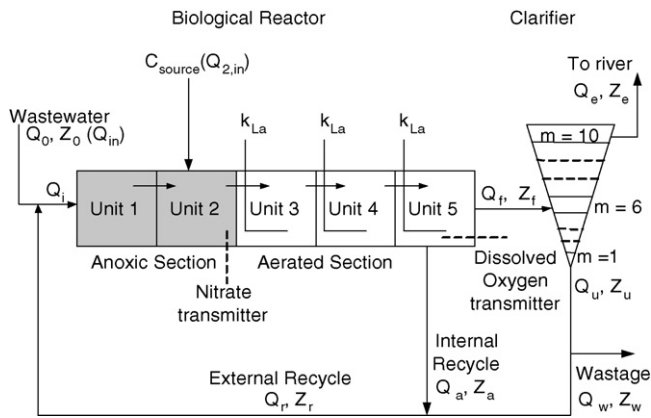


Fig. 1. The BSM1 benchmark layout.

handling is necessary to respect the environmental norms. According to [20] who performed a real application, the first objective of the supervisory control strategy is to optimize the existing plant capacity, to improve the process quality and effluent quality with minimum cost. Thus, MPC is a very suitable control technology for this application [1] in order to keep the plant running effectively and to meet the discharge quality standards. MPC has also been implemented on several complex nonlinear environmental systems [26,11,27]. [5] implemented dissolved oxygen control of the activated sludge system using MPC. However, their works have been focused on the dissolved oxygen control rather than based on the assumption of a multivariable control problem. Rather than focusing on a single problem such as dissolved oxygen control or nitrate control, this study aims at considering the wastewater treatment plant in a large multivariable frame subject to environmental and operational constraints. However, it is limited to the wastewater treatment plant and does not consider the sewer systems [2]. In this paper, three different kinds of MPC strategies are used: DMC algorithm without constraints which represents the first generation of the MPC algorithms, a QDMC version with hard linear constraints which is considered to be a representative of the second generation of the MPC algorithms, a NLMPC version with hard constraints on the inputs and soft constraints on the outputs. Furthermore, as the influent flow rate and ammonium concentration can be considered as measured disturbances, feedforward has been added to previous feedback MPC controls in an effort to compensate these large influent time variations. Thus, feedback and feedforward controls will also be compared.

## 2. Benchmark of wastewater treatment plants

The International Association of Water Quality (IAWQ) and COST (European Cooperation in the field of Scientific and Technical Research) 624 group have established acknowledged models representing the behavior of wastewater treatment plants that can be used to test estimation, diagnostic and control strategies. COST 624 group published a benchmark [28,29,13], also available on the following web site: <http://www.ensic.inpl-nancy.fr/COSTWWTP/>.

The simulated wastewater treatment plant possesses a series of five reactors, the first two ones being mixed and non-aerated, the three following ones being simply aerated; this group is followed by a secondary settler (Fig. 1). Two recycle streams complete the process. The model of the biological process is Activated Sludge Model #1 (ASM1) of IAWQ and includes 13 components and 8 reaction processes. Typical feed disturbances for dry, stormy or rainy weather are available as representative files of 14 days with a sampling period of 15 min. Performance criteria have been established concerning the effluent quality, constraints corresponding to the

operating norms are imposed on the effluents and operating costs are proposed.

The importance of wastewater treatment control is emphasized by many authors. The process dynamics is complex. The choice of the control structure is important. Moreover, operating constraints and the nonlinear behavior of the process make the process control problem very attractive for performing multivariable algorithms such as MPC ones.

## 3. Model predictive control

The MPC strategy was first introduced by [24]. Very soon after, DMC was published [30] and implemented at Shell as a multivariable computer control algorithm. Different forms of MPC are possible, based on step or impulse responses, or under state-space forms. The principle is well known in the literature [31–33] and only a brief description with main features is presented here. DMC [30] simply minimizes a quadratic criterion in absence of constraints. Being an extension of DMC, QDMC [34] minimizes the same quadratic criterion in the presence of linear constraints. In the present study, different forms of feedback MPC have been implemented [35]: DMC, QDMC and NLMPC. Furthermore, a modified version of QDMC incorporating feedforward has been developed.

In the present study, truncated step responses are used to represent the linear model of the process. Typically, they can be obtained from open loop simulations if a nonlinear model of the process based on first principles is available or from response measurements to variations of manipulated inputs in an existing plant.

In DMC, a quadratic criterion based on the errors between the estimated outputs  $\hat{y}(k+i|k)$  and the references  $y^{\text{ref}}(k+i)$  over the prediction horizon  $H_p$  is defined as

$$J = \sum_{i=1}^{H_p} (\hat{y}(k+i|k) - y^{\text{ref}}(k+i))^2 \quad (1)$$

and is minimized with respect to the variation  $\Delta u(k)$  of the input considered over a control horizon  $H_c$  which is much smaller than  $H_p$ . The output prediction  $\hat{y}(k+l|k)$  means a predicted controlled outputs for the future sampling instant  $k+l$ , performed at the current instant  $k$ . It can be decomposed as a steady-state term, a term as effect of past inputs, a term as effect of future inputs, and a term of disturbances.

$$\hat{y}(k+l|k) = y_{ss} + \sum_{i=l+1}^{H_m-1} h_i \Delta u(k+l-i) + h_{H_m}(u(k+l-H_m) - u_{ss}) + \sum_{i=1}^l h_i \Delta u(k+l-i) + \hat{d}(k+l|k) \quad (2)$$

where  $H_m$  is the model horizon which should be larger or equal to the prediction horizon.

The output prediction  $y^*(k+l|k)$  based on past inputs is defined as

$$y^*(k+l|k) = y_{ss} + \sum_{i=l+1}^{H_m-1} h_i \Delta u(k+l-i) + h_{H_m}(u(k+l-H_m) - u_{ss}) \quad (3)$$

Note that the steady-state terms  $u_{ss}$  and  $y_{ss}$  do not intervene if deviation variables are used. Finally, the vector of the output prediction  $\hat{y}(k+l|k)$  based on past and future inputs is related to the vector of the output prediction  $y^*(k+l|k)$  based on past inputs, to the vector

of inputs  $\Delta u(k)$ , and to the vector of predicted disturbances by

$$\begin{bmatrix} \hat{y}(k+1|k) \\ \vdots \\ \hat{y}(k+H_p|k) \end{bmatrix} = \begin{bmatrix} y^*(k+1|k) \\ \vdots \\ y^*(k+H_p|k) \end{bmatrix} + \mathcal{A} \begin{bmatrix} \Delta u(k) \\ \vdots \\ \Delta u(k+H_c-1) \end{bmatrix} + \begin{bmatrix} \hat{d}(k+1|k) \\ \vdots \\ \hat{d}(k+H_p|k) \end{bmatrix} \quad (4)$$

where  $\mathcal{A}$  is the dynamic matrix whose elements are taken from the step response coefficients  $h_i$  of the plant outputs to the manipulated inputs [35]. For a single input–single output system, the dynamic matrix  $\mathcal{A}$  with the dimension of  $H_p \times H_c$  is equal to

$$\mathcal{A} = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ h_2 & h_1 & & \vdots \\ \vdots & \vdots & \ddots & \\ h_{H_c} & h_{H_c-1} & \cdots & h_1 \\ \vdots & \vdots & & \vdots \\ h_{H_m} & h_{H_m-1} & \cdots & h_{H_m-H_c+1} \\ \vdots & \vdots & & \vdots \\ h_{H_m} & h_{H_m} & \cdots & h_{H_m} \end{bmatrix} \quad (5)$$

For a multi-input–multi-output system of dimension  $n_u \times n_y$ , the dynamic matrix becomes

$$\mathcal{A} = \begin{bmatrix} \mathcal{A}_{11} & \cdots & \mathcal{A}_{1n_u} \\ \vdots & \mathcal{A}_{ij} & \vdots \\ \mathcal{A}_{n_y1} & \cdots & \mathcal{A}_{n_y n_u} \end{bmatrix} \quad (6)$$

where  $i$  refers to the controlled output  $y_i$  and  $j$  to the manipulated input  $u_j$ . In classical feedback MPC, it is assumed that future disturbances are unknown, therefore the disturbance value is assumed to be constant over the prediction horizon.

$$\hat{d}(k+l|k) = \hat{d}(k|k) = y^m(k) - y^*(k|k) \quad \forall l = 1, \dots, H_p \quad (7)$$

Defining the error  $e(k)$ , a linear system can be written

$$\begin{bmatrix} y^{\text{ref}}(k+1) - y^*(k+1|k) - \hat{d}(k|k) = e(k+1) \\ \vdots \\ y^{\text{ref}}(k+H_p) - y^*(k+H_p|k) - \hat{d}(k|k) = e(k+H_p) \end{bmatrix} = \mathbf{e}(k+1) = \mathcal{A} \Delta \mathbf{u}(k) \quad (8)$$

In absence of constraints, the least-squares solution of this DMC optimization problem yields the future input move vector:

$$\Delta \mathbf{u}(k) = (\mathcal{A}^T \mathcal{A})^{-1} \mathcal{A}^T \mathbf{e}(k+1) \quad (9)$$

Only the first calculated input  $\Delta u(k)$  will be implemented at time  $k$ .

An attractive feature of MPC is the possibility to handle constraints. QDMC introduces a modification of the quadratic criterion as the sum of a performance term and an energy term. Furthermore, typical hard constraints are considered, including those affecting the manipulated variables:

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max} \quad (10)$$

and their moves

$$\Delta \mathbf{u}_{\min} \leq \Delta \mathbf{u} \leq \Delta \mathbf{u}_{\max} \quad (11)$$

These constraints correspond to valve positions and velocities. They cannot be violated. All these constraints can be gathered as a system of linear inequalities incorporating the dynamic information concerning the projection of constraints:

$$\mathbf{B} \Delta \mathbf{u}(k) \leq \mathbf{C}(k+1) \quad (12)$$

In presence of hard constraints (10) and (11), the QDMC problem can be thus formulated as quadratic programming, such as

$$\min_{\Delta \mathbf{u}(k)} J = \left[ \frac{1}{2} \Delta \mathbf{u}(k)^T \mathbf{H} \Delta \mathbf{u}(k) - \mathbf{g}(k+1)^T \Delta \mathbf{u}(k) \right] \quad (13)$$

subjects to constraints (10) and (11).  $\mathbf{H}$  is the Hessian matrix (in general fixed) which is equal to

$$\mathbf{H} = \mathcal{A}^T \mathbf{\Gamma}^T \mathbf{\Gamma} \mathcal{A} + \mathbf{\Lambda}^T \mathbf{\Lambda} \quad (14)$$

where  $\mathcal{A}$  is the dynamic matrix,  $\mathbf{\Gamma}$  is a diagonal matrix of weights for the outputs,  $\mathbf{\Lambda}$  is a diagonal matrix of weights for the inputs, and  $\mathbf{g}$  is the gradient vector which is equal to

$$\mathbf{g}(k+1) = \mathcal{A}^T \mathbf{\Gamma}^T \mathbf{\Gamma} \mathbf{e}(k+1) \quad (15)$$

This quadratic programming problem can be solved efficiently by available numerical subroutines.

Real controlled processes are usually nonlinear, and a linear feedback controller designed for a vicinity of the assumed operating point is not always sufficient. In the 1980s, attempts to formulate MPC algorithms for nonlinear process models appeared, especially within the community connected with development and applications of the DMC algorithm in chemical industries. In the case of the benchmark of the wastewater treatment plant, it is not authorized to directly consider the full nonlinear model for control. Therefore, a nonlinear model can be used only after it is obtained independently from the true model of the plant, for example by some reduction technique obtained from plant responses, like in [36]. Two internationally accepted process models were chosen in the simulation benchmark. The IAWQ's Activated Sludge Model #1 (ASM1) was chosen as the biological process model [37] and the double-exponential settling velocity function of Takács et al. [38] was chosen as a fair representation of the settling process. In the present article, only the step responses already used for DMC and QDMC techniques were retained for all other simulations. When constraints affect the output variables, it is safer to consider them as soft constraints rather than hard constraints to avoid the failure of the nonlinear optimization procedure. Thus, soft constraints can be violated at certain times. The criterion is very similar to Eq. (13) of the QDMC except that a penalty function with respect to the predicted outputs is added to the criterion as

$$0.5 w_{py} [(|y_{\max} - \hat{y}| - (y_{\max} - \hat{y}))^2 + (|\hat{y} - y_{\min}| - (\hat{y} - y_{\min}))^2] \quad (16)$$

The penalty term is zero when the predicted output  $\hat{y}$  is between the output bounds  $y_{\min}$  and  $y_{\max}$  and the penalty function penalizes the deviations above the maximum and below the minimum allowed to the controlled outputs. Thus, the resulting problem is non-quadratic and necessitates a nonlinear optimization. In this case, the nonlinear optimization code NLPQL [39] is used. A drawback of this technique is that the robustness of the code is less guaranteed and the computation is much slower than with the normal quadratic programming. Consequently, this MPC strategy is called NLMPC because of the nonlinear optimization involved, even if the model of the process remains the same dynamic matrix as for DMC and QDMC. The potential of NLMPC is larger than the present use and a nonlinear model of the process could have been implemented.

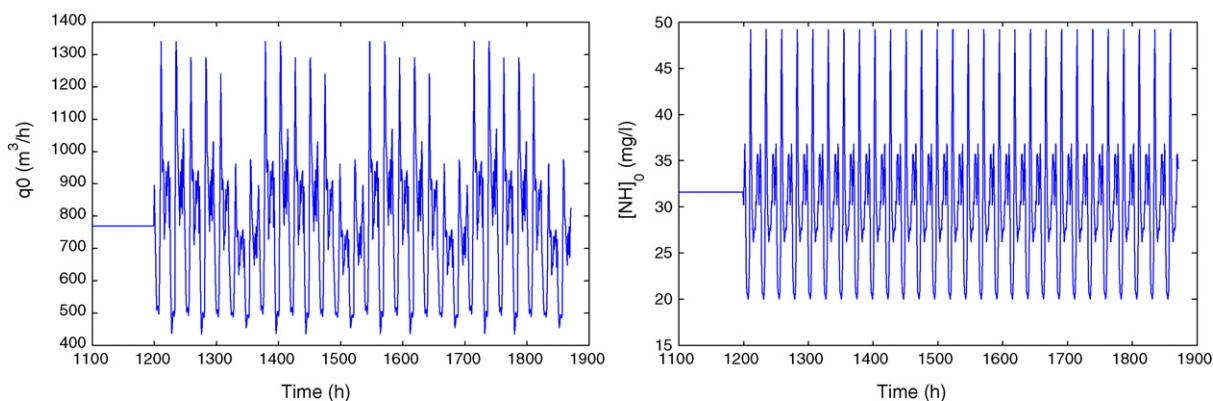


Fig. 2. Disturbances of influent flow rate and ammonium concentration for a dry weather influent file.

The different MPC codes: DMC without constraints, QDMC with hard linear constraints, NLMPC with hard and soft constraints, MPC with feedforward, have been developed as general Fortran programs which are able to take into account most types of situation, including open-loop inverse responses, hard constraints on the inputs and on their variations, soft constraints on the outputs, feedforward effects and consider any number of inputs and outputs.

#### 4. Influent load

Simulated influent data are available in three 2-week files derived from real operating data [40,41]. The files were generated to represent three different weather situations. The sampling period is 0.25 h. The file which was used in the present study was only representative of a dry weather period. The file exhibits characteristic diurnal variations in flow and component concentrations. Also incorporated in the file is a substantial (20%) decrease in flow and load during the “week-end”. Fig. 2 shows two variables from the dry weather file (flow rate and ammonium concentration).

Two typical situations were studied: in the absence and in the presence of measured disturbances. The disturbances were given by data files representing typical real situations of normal dry weather with flow rate and concentration fluctuations. It must be noticed that the variations of the disturbances around a mean value are considerable and present more or less a periodical aspect based on a 1-day period.

#### 5. Control strategy

As a multivariable input–output system, the control system is described as following.

##### 5.1. Bounds, measurements, outputs, inputs and disturbances

The Linear DMC, QDMC and NLMPC strategies have been tested to maintain the effluent qualities within regulation-specified limits, whatever the variations of the incoming wastewater. The limits on ammonium concentration, suspended solids concentration, BOD<sub>5</sub> (oxygen demand of biodegradable pollutants over a 5-day period),

COD (chemical oxygen demand) and total nitrogen concentration are given in Table 1.

The two measured variables are the dissolved oxygen concentration in the last unit of the bioreactor and the nitrate level in the second non-aerated unit.

The five controlled variables are the ammonium concentration, the concentration of suspended solids, the BOD<sub>5</sub>, the COD and the total nitrogen in the effluents.

The seven manipulated variables are the internal recycle flow rate  $q_a$ , the external recycle flow rate  $q_r$ , the wastage flow rate  $q_w$ , the mass transfer coefficients in the third, fourth and fifth aerated tanks respectively  $kla_3$ ,  $kla_4$ ,  $kla_5$ , the carbon source supplementation flow rate  $q_{2in}$ . The mass transfer coefficient corresponds to the efficiency of aeration in a given aerated tank.

In order to improve the controller performance, two measurable disturbances have been considered: the influent flow rate  $q_0$  and the influent ammonium concentration  $[NH]_0$ . The feedforward controller (Section 6.5) will specifically make use of these measurements.

##### 5.2. Constraints on the manipulated inputs and outputs

The limits of the effluent quality indicators are transformed into fixed set points, lower than the bounds (Table 1) with the final objective in mind that, by this way, these upper bounds will be respected most of the time. The ammonium concentration and total nitrogen are the two most sensitive outputs and react very strongly to disturbances. The discussion will focus mainly on ammonium concentration as the total nitrogen concentration in general varies in a similar way. Their weights  $\Gamma_{ii}$ , referring to Eq. (14), are larger than other weights in the MPC strategies. Saturations concerning the manipulated inputs are given in Table 2.

##### 5.3. Horizons and weights

The prediction horizon  $H_p$ , control horizon  $H_c$  and model horizon  $H_m$  are taken respectively as 45 sampling periods, 3 sampling periods and 48 sampling periods ( $T_s = 0.5$  h) which represents 1 day behavior (strategy DMC-FB-1 of Table 4 for DMC). The control

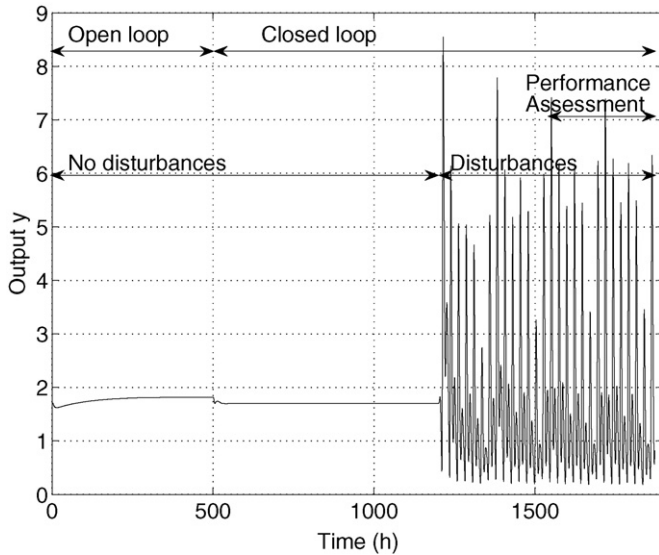
Table 1  
Set points, bounds, weights for the controlled outputs.

Controlled output $y$	Upper bound	Set point	Weight $\Gamma_{ii}$	Weight $w_{py}$ in penalty function
Ammonium $[NH]$ (mg/l)	4	1.7	90	1000
Total nitrogen $[N]_{tot}$ (mg/l)	18	14	50	1000
Suspended solids $[SS]$ (mg/l)	30	12.5	5	1000
BOD <sub>5</sub> (mg/l)	10	2.7	5	1000
COD (mg/l)	100	47.5	5	1000

**Table 2**

Saturation limits, weights for the manipulated inputs.

Manipulated input $u$	Minimum value	Maximum value	Weight $\Lambda_{ii}$
Internal recycle flow rate, $q_a$ (m <sup>3</sup> /h)	1000	4000	100
External recycle flow rate, $q_r$ (m <sup>3</sup> /h)	200	1000	100
Wastage flow rate, $q_w$ (m <sup>3</sup> /h)	10	40	100
Oxygen mass transfer coefficient 3rd unit, $kla_3$ (l/h)	0	15	100
Oxygen mass transfer coefficient 4th unit, $kla_4$ (l/h)	0	15	100
Oxygen mass transfer coefficient 5th unit, $kla_5$ (l/h)	0	15	100
Carbon source supplementation flow rate, $q_{2in}$ (m <sup>3</sup> /h)	0	0.2	100

**Fig. 3.** Different phases during the simulation sequence. The plotted output is the effluent ammonium concentration.

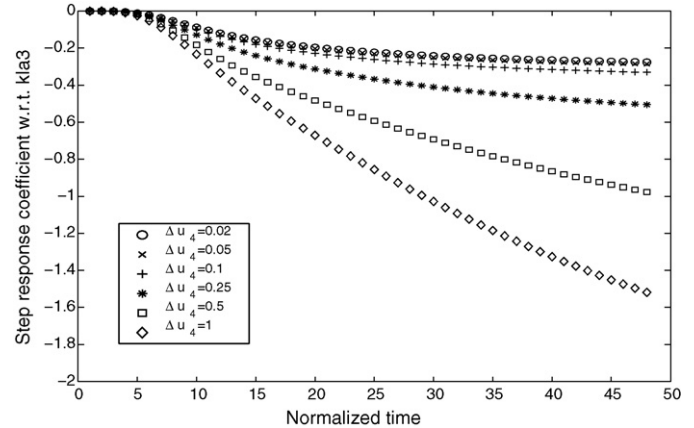
horizon was taken fixed equal to  $H_c = 3$  as a higher value has little influence on the control performance. Other choices of prediction horizon and model horizon have been tested (Table 4). For  $H_p = 93$  and  $H_m = 96$  (2 days), there was no improvement and little modification of the control performance (strategy DMC-FB-2). For  $H_p = 141$  and  $H_m = 144$  (3 days), the control was very much degraded (strategy DMC-FB-3). QDMC showed the same trend (strategies QDMC-FB-1 and QDMC-FB-2). Due to the estimated time constants of the open loop responses,  $H_p = 45$  and  $H_m = 48$  seemed a good compromise and was retained for all other simulations.

In practice, the inputs and outputs intervening in the criterion are normalized by their steady-state values to render the choice of the weights easier.  $\Gamma_{ii}$  are given in Table 1, all  $\Lambda_{ii}$  are identical and equal to 100 (Table 2). The weights intervening in the penalty function which force the process to respect the soft constraints on the outputs are very large with respect to the other weights so

**Table 3**

Steady-state value, step magnitude and duration for the manipulated inputs and measured disturbances.

Manipulated input $u$	Steady-state value	Step magnitude	Step duration (h)
Internal recycle flow rate (m <sup>3</sup> /h)	2306	+10%	24
External recycle flow rate (m <sup>3</sup> /h)	769	+10%	24
Wastage flow rate (m <sup>3</sup> /h)	16	+10%	24
Oxygen mass transfer coefficient 3rd unit (l/h)	10	-4%	24
Oxygen mass transfer coefficient 4th unit (l/h)	10	-4%	24
Oxygen mass transfer coefficient 5th unit (l/h)	3.5	+4%	24
Carbon source supplementation flow rate (m <sup>3</sup> /h)	0.083	+10%	24
Measured disturbance $d$		Step magnitude	Step duration (h)
Influent flow rate $q_0$ (m <sup>3</sup> /h)		+10%	24
Influent ammonium concentration [NH] <sub>0</sub> (mg/l)		+10%	24

**Fig. 4.** Step response coefficients of effluent ammonium concentration to oxygen mass transfer coefficient in the third tank for various amplitudes of the oxygen mass transfer coefficient.

that the penalty function becomes preponderant in the criterion (Table 1).

## 6. Simulation results and discussion

All the results have been obtained using the benchmark Fortran implementation described in [22] except that the differential equations were integrated with a 5th-order Runge-Kutta routine (fixed integration step = 0.01 h). DMC, QDMC control and NL MPC were also used as independent Fortran codes.

Fig. 3 presents the simulation sequence which has been used in the present work: an open loop stabilization period under constant inputs (period 1 = [0, 500] h), a closed loop stabilization period in absence of disturbances (period 2 = [500, 1200] h), two closed loop dynamic periods in presence of disturbances, in the present case with the dry weather disturbance file (period 3 = [1200, 1536] and period 4 = [1536, 1872], each representing 14 days). Period 3 is considered as a dynamic stabilization period and period 4 is used for performance evaluation. In fact, the same controller is operated during the closed loop phase, i.e. from time  $t = 500$  h until the end.

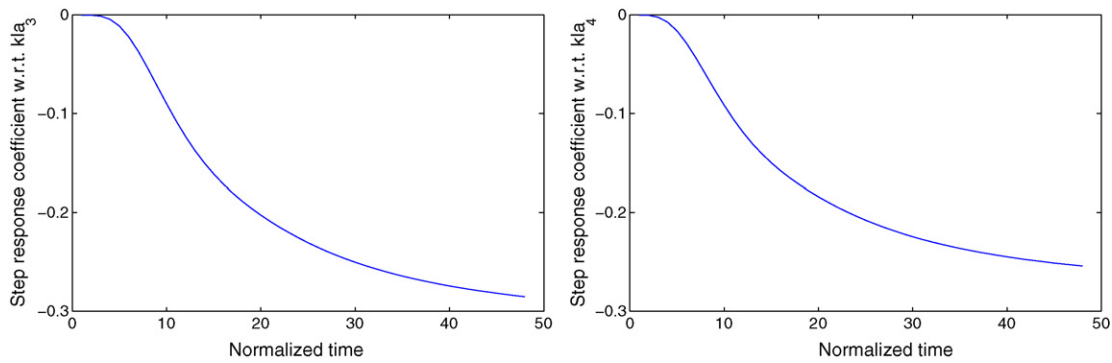


Fig. 5. Step response coefficients of effluent ammonium concentration to oxygen mass transfer coefficients in the third (left) and fourth (right) tanks.

### 6.1. Open loop responses of system

Open loop responses have been obtained from step variations of the manipulated inputs and measured disturbances around the steady states. For a given output  $y_i$  ( $i = 1, \dots, n_y$ ), the step response with respect to the manipulated inputs is in fact represented by means of the step response coefficients  $h_{j,i}(k)$  ( $j = 1, \dots, n_u$ ) at time  $k$  ( $k > 0$ ), i.e.:

$$h_{j,i}(k) = \frac{y_i(k) - y_i^{ss}}{u_j^\infty - u_j^{ss}} \quad (17)$$

and similarly, with respect to the measured disturbances  $d_{m,j}$  ( $j = 1, \dots, n_d$ ) as

$$h'_{j,i}(k) = \frac{y_i(k) - y_i^{ss}}{d_{m,j}^\infty - d_{m,j}^{ss}} \quad (18)$$

where the superscript *ss* refers to the steady-state value and  $\infty$  to the asymptotic value at the end of the step response. These open loop step responses will constitute the linear model of the process under the form of the dynamic matrix which is used in MPC. In an existing plant, they would be replaced by the identified responses fitted to the actual input–output responses of the plant [42] which are obtained after system excitation either in open loop or in closed loop. Thus, in no case, the nonlinear benchmark model is used directly for MPC. It is only used to simulate an existing plant.

The step magnitude and duration of each manipulated input and measured disturbance are given in Table 3. The step responses were obtained from  $t = 1200$  h imposing step signals on each input  $u_j$  ( $j = 1, \dots, n_u$ ) and each measured disturbance  $d_{m,j}$  ( $j = 1, \dots, n_d$ ).

The concentration of ammonium in the effluent is one of the two most sensitive outputs together with the total nitrogen con-

centration so that the discussion will focus on it. In the case of a linear system, the coefficients of the step response are independent of the amplitude of the input step. In the present case, in order to demonstrate the nonlinearity of the process on an example, the influence of the amplitude of the input step on the step response coefficient is shown in the case of the step response of effluent ammonium concentration to oxygen mass transfer coefficient in the third tank (Fig. 4). It follows that the choice of the amplitude of the input step may have a large influence on the open loop model of the process and consequently on the performance of MPC so that this choice has a somewhat arbitrary character. It has been found that the step responses corresponding to the influences of the internal recycle flow rate  $q_a$ , the external recycle flow rate  $q_r$ , and the wastage flow rate  $q_w$  on ammonium concentration (not represented here) are lower by at least one order of magnitude than the responses displayed in Figs. 5 and 6. The responses to the oxygen mass transfer coefficients in the third to fifth aerated tanks show relatively similar behaviors, however the response of the fifth tank is approximately two to three times larger than that of the previous tanks. The response to the carbon source supplementation flow rate  $q_{2in}$  displays an important influence of this variable. For finite steps of all manipulated inputs except for external recycle flow rate and wastage flow rate, in general, the responses of the plant are classical and stable. However, because of the direct influence of the concerned manipulated input on the studied output, the response (not shown) of the effluent ammonium concentration to the step of external recycle flow rate  $q_r$  possesses an inverse response of small magnitude. This influence of inverse response is avoided in the later MPC control system by considering that an inverse response is close to a response of a delayed system and by neglecting the first transient inverse behavior. For the step of wastage flow rate, the low response of effluent ammonium concentra-

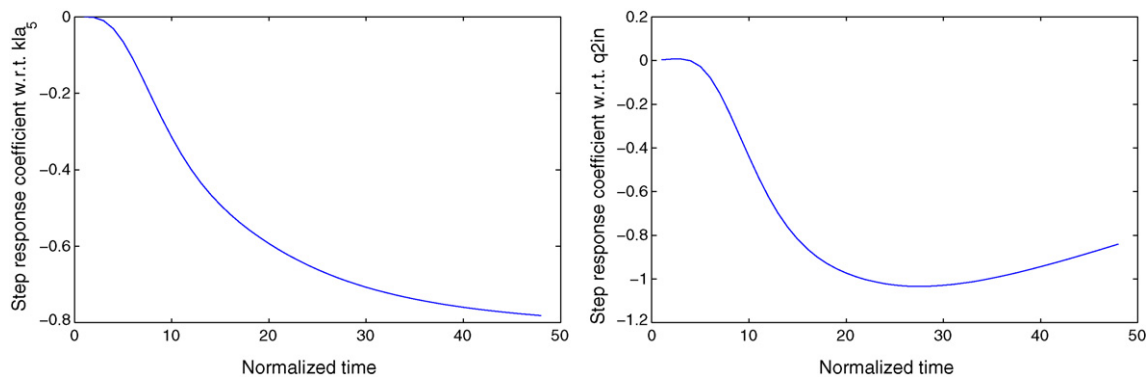
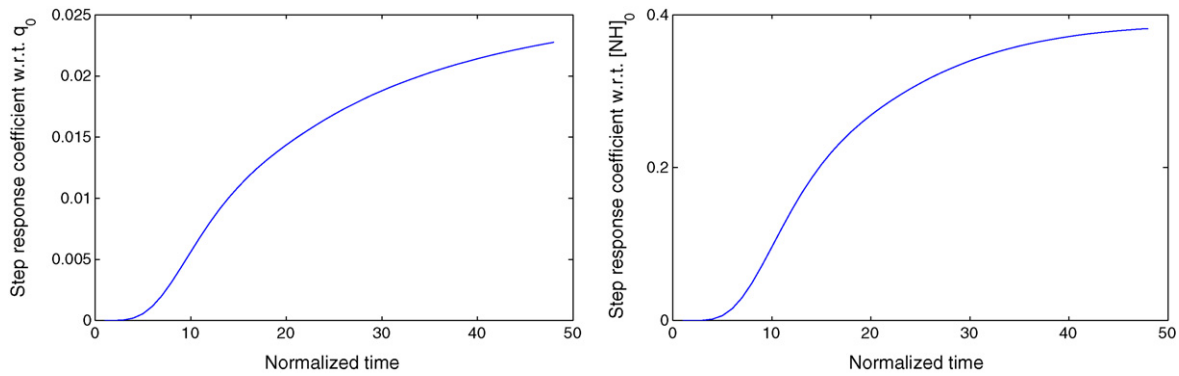


Fig. 6. Step response coefficient of effluent ammonium concentration to oxygen mass transfer coefficient in the fifth tank (left) and to carbon source supplementation flow rate (right).



**Fig. 7.** Step response coefficient of effluent ammonium concentration to the disturbance of influent flow rate (left) and to the disturbance of influent ammonium concentration (right).

tion is apparently unstable at least during the period of time considered.

In order to improve the controller performance, the characteristics of two measurable disturbances have been considered. As the effluent ammonium concentration is one of two most sensitive outputs, the disturbance of influent ammonium concentration  $[\text{NH}]_0$  plays an important role and is considered as a measurable variable of the plant. The second measured disturbance is the influent flow rate  $q_0$ . It is a good indication of changes in the wastewater characteristics which are related to human activity. In Fig. 7, the step response coefficients  $h'$  of effluent ammonium concentration to the measured disturbances of the influent flow rate  $q_0$  and the influent ammonium concentration  $[\text{NH}]_0$  are represented. It is clear that the influence of the influent ammonium concentration is larger by one order of magnitude than the influence of the influent flow rate. It has also been found that the shapes of the responses are dependent on the amplitudes of the respective disturbances, which again demonstrates the nonlinear behavior of the process. In practice, large disturbances of influent flow rate  $q_0$  and influent ammonium concentration  $[\text{NH}]_0$  (Fig. 2) perturb the process, with respective relative variations of  $[-46\%, +74\%]$  and  $[-31\%, +56\%]$  around the steady-state value. Consequently, the magnitudes of the disturbance steps used to obtain the values of the step response coefficients were chosen large and were equal to 10% (Table 3). The responses of effluent ammonium concentration to a finite step of both the influent flow rate and the influent ammonium concentration can be considered as second order responses with delay (Fig. 7) or even close to first order responses with delay.

## 6.2. Summary of different control strategies

Many different simulations have been performed with different feedback MPC controllers or combined feedforward-feedback controllers. Some of them have been retained to show quantitatively their influences on different mean outputs and criteria (Table 4). They are summarized in the following list.

DMC-FB-1: DMC without feedforward, with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ .

DMC-FB-2: DMC without feedforward, with  $H_m = 96$ ,  $H_p = 93$ ,  $H_c = 3$ .

DMC-FB-3: DMC without feedforward, with  $H_m = 144$ ,  $H_p = 141$ ,  $H_c = 3$ .

QDMC-FB-1: QDMC without feedforward, with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ .

QDMC-FB-2: QDMC without feedforward, with  $H_m = 96$ ,  $H_p = 93$ ,  $H_c = 3$ .

NLMPC-FB: NLMPC without feedforward, with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ .

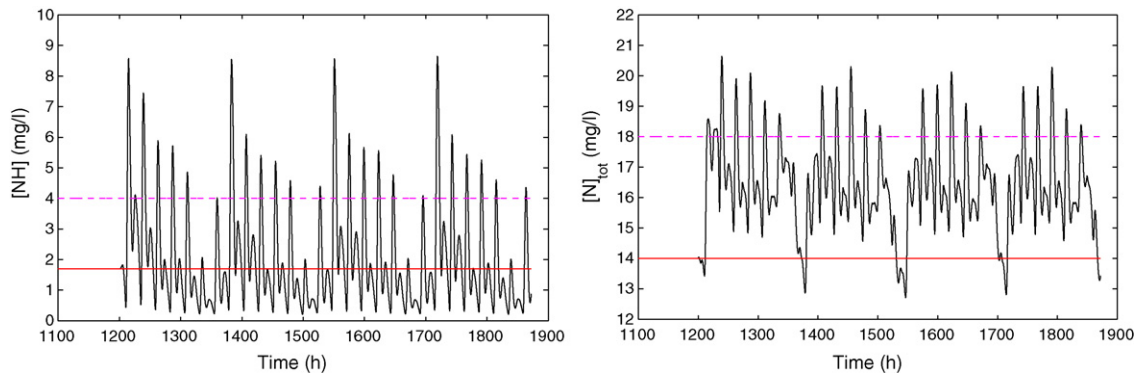
DMC-FF- $q_0$ -1: DMC with feedforward on  $q_0$ , with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ . The disturbance is considered on a horizon equal to 3 and is not filtered.

DMC-FF- $q_0$ -2: DMC with feedforward on  $q_0$ , with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ . The disturbance is considered on a horizon equal to 3 and is filtered by a forgetting factor ( $f = 0.98$ ) filter.

QDMC-FF- $q_0$ : QDMC with feedforward on  $q_0$ , with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ . The disturbance is considered on a horizon equal to 3 and is filtered by a forgetting factor ( $f = 0.98$ ) filter.

**Table 4**  
Results obtained with different MPC strategies.

Strategy	$[\text{NH}]$ (mg/l)	$[\text{N}]_{\text{tot}}$ (mg/l)	$[\text{SS}]$ (mg/l)	$\text{BOD}_5$ (mg/l)	$\text{COD}$ (mg/l)	Effluent quality (kg pollution units/day)	Pumping energy (kWh/day)	Aeration energy (kWh/day)	ISWAE criterion ( $\text{g m}^{-3}$ days)
DMC-FB-1	1.76	14.64	14.18	2.88	49.77	214.6	409.4	4326	$1.54 \times 10^3$
DMC-FB-2	1.82	14.48	14.35	2.90	50.00	215.3	433.2	4285	$1.58 \times 10^3$
DMC-FB-3	13.08	22.01	10.07	2.49	44.59	497.61	195.2	6398	$2.67 \times 10^4$
QDMC-FB-1	1.94	16.25	13.00	2.78	48.20	226.6	405.8	5339	$1.79 \times 10^3$
QDMC-FB-2	1.91	15.22	13.55	2.82	48.94	219.93	430.8	4627	$1.60 \times 10^3$
NLMPC-FB	2.01	14.25	14.19	2.87	49.79	217.3	468.5	4125	$1.65 \times 10^3$
DMC-FF- $q_0$ -1	3.06	16.49	12.55	2.73	47.62	254.3	262.3	5684	$2.60 \times 10^3$
DMC-FF- $q_0$ -2	1.72	14.73	14.36	2.89	50.00	214.79	404.4	4631	$1.63 \times 10^3$
QDMC-FF- $q_0$	1.92	15.77	13.06	2.78	48.28	222.5	399.7	5259	$1.68 \times 10^3$
DMC-FF- $[\text{NH}]_0$ -1	7.03	20.56	10.99	2.59	45.66	371.6	155.8	6368	$6.58 \times 10^3$
DMC-FF- $[\text{NH}]_0$ -2	1.30	14.51	14.60	2.93	50.31	204.48	394.0	4720	$1.57 \times 10^3$
DMC-FF- $q_0$ - $[\text{NH}]_0$	1.26	14.64	14.75	2.93	50.50	204.98	410.8	5022	$1.58 \times 10^3$



**Fig. 8.** Outputs in last aerated unit in case of QDMC without feedforward action: ammonium concentration (left) and total nitrogen concentration (right). The continuous line is the set point. The dashed line is the upper bound.

DMC-FF-[NH]<sub>0</sub>-1: DMC with feedforward on [NH]<sub>0</sub>, with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ . The disturbance is considered on a horizon equal to 3 and is not filtered.

DMC-FF-[NH]<sub>0</sub>-2: DMC with feedforward on [NH]<sub>0</sub>, with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ . The disturbance is considered on a horizon equal to 3 and is filtered by a forgetting factor ( $f = 0.98$ ) filter.

DMC-FF- $q_0$ -[NH]<sub>0</sub>: DMC with feedforward on  $q_0$  and [NH]<sub>0</sub>, with  $H_m = 48$ ,  $H_p = 45$ ,  $H_c = 3$ . The disturbances are considered on a horizon equal to 3 and are filtered by a forgetting factor ( $f = 0.98$ ) filter.

In the present case, the parameters were tuned on the basis of the experience gained from the simulations and from the tuning rules presented in [32]. In this highly multivariable system where many couplings are present, the tuning is quite delicate even if we have tried to work with dimensionless inputs and outputs and to follow the evolution of the criteria with time.

### 6.3. Control statistics

Statistics have been calculated concerning the mean effluent concentrations (Table 4) during the assessment period ([1536, 1872] h) with the sampling period of 0.5 h. The outputs indicated in Table 4 are given with respect to their mean value. The main outputs to be followed are the ammonium and total nitrogen concentrations. Their relative values should be discussed together with the value of the criterion taken as the integral of the sum of weighted absolute errors (ISWAE) which is defined with respect to the definition of the effluent quality and are given in the same table. The ISWAE criterion is defined as

$$\text{ISWAE} = \int_0^{t_{\text{obs}}} \{30|[NH] - [NH]_{sp}| + 10|[N]_{\text{tot}} - [N]_{\text{tot},sp}| + 2|[SS] - [SS]_{sp}| + 2|[BOD] - [BOD]_{sp}| + |[COD] - [COD]_{sp}|\} dt \quad (19)$$

where  $sp$  refers to the set point. Furthermore, three statistical indicators recommended in a recent version of the benchmark have been used. The effluent quality index EQI is given as

$$\text{EQI} = \frac{1}{1000 \Delta t_{\text{obs}}} \int_0^{t_{\text{obs}}} \{30[NH] + 10[N]_{\text{tot}} + 2[SS] + 2[BOD]_5 + [COD]\} q_e dt \quad (20)$$

where  $[0, t_{\text{obs}}]$  is the interval of observation, here [1536, 1872] h.  $q_e$  is the effluent flow rate. The pumping energy  $E_p$  is

$$E_p = \frac{1}{\Delta t_{\text{obs}}} \int_0^{t_{\text{obs}}} \{0.004 q_a + 0.008 q_r + 0.05 q_w\} dt \quad (21)$$

The aeration energy  $E_a$  is

$$E_a = \frac{[O]_{\text{sat}}}{1800 \Delta t_{\text{obs}}} \int_0^{t_{\text{obs}}} \{V_3 kla_3 + V_4 kla_4 + V_5 kla_5\} dt \quad (22)$$

where  $[O]_{\text{sat}}$  is the saturated oxygen concentration.  $V_i$  is the volume of the  $i$ th tank.

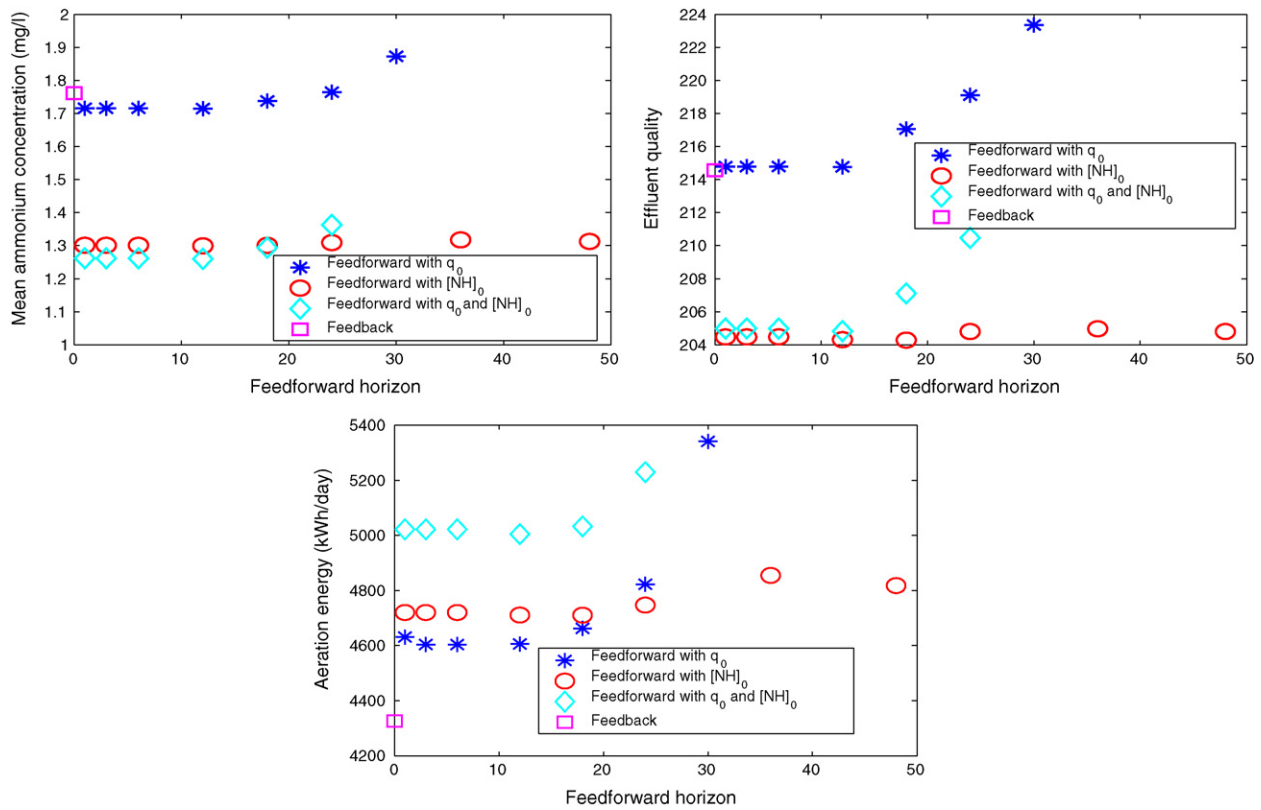
In general, it is found that the effluent ammonium and total nitrogen concentrations vary in the same way as the effluent quality.

### 6.4. In absence of feedforward controller

In absence of disturbances, in the time interval [500, 1200] h, the feedback controller performs very well, even under the form of DMC without any constraint, and drives the outputs towards their set points in a way similar to classical processes. When the considerable disturbances which are present in a wastewater treatment plant occur after  $t = 1200$  h, the task of the controller becomes much more difficult. During this period, the outputs are very sensitive to the strong amplitude of the disturbances. Due to the large range of time constants inherent in the activated sludge process, the occurrence of disturbances causes serious deterioration of the effluent quality (Fig. 8). In this period, the moves of the manipulated inputs which are calculated by DMC used without any constraint are so large that they go beyond the saturation limits given in Table 2. Consequently, even for DMC, simple constraints valid only for the last calculated input vector are imposed. They are sufficient to avoid the explosion of the multivariable controller. QDMC differs from this slightly modified DMC as the saturation limits are used as constraints for all the values of the future inputs, not only the last calculated value. Furthermore, QDMC incorporates an energy part in the quadratic criterion. Due to the large range of time constants inherent in the activated sludge process, the appearance of disturbance causes deterioration of the effluent quality.

The statistical results show little difference between DMC (strategy DMC-FB-1), QDMC (strategy QDMC-FB-1) and NLMPC (strategy NLMPC-FB), in absence of feedforward control action. The best performance for total nitrogen is obtained by the NLMPC strategy (NLMPC-FB) but the worst performance for ammonium is also by the NLMPC strategy. Other weights for the penalty function might have resulted in a different way. As all the values or a given variable are contained in a narrow domain, their differences cannot be considered as strongly significant and sometimes an advantage with respect to one of them is counterbalanced by a drawback on another one. The term of aeration energy is always much larger than the pumping energy. The effluent quality is well





**Fig. 9.** Comparison of feedforward strategies for different values of the feedforward horizon. Top, left: mean ammonium concentration. Top, right: effluent quality. Bottom: aeration energy.

related to the tendency with respect to the ammonium concentration.

### 6.5. In presence of feedforward controller

In order to improve the multivariable controller performance, a model-based feedforward controller using the information on measured disturbances has been added to the model predictive controller. This results in a feedforward-feedback controller, simply noted as feedforward controller in the following. It must be noted that the feedforward controller has never been used alone. Two disturbances have been considered for the different feedforward controllers: the influent ammonium concentration and the influent flow rate.

In the MPC control strategy used here, the measured disturbances are considered in a very similar way to the past manipulated inputs as far as they are measured and their influences on the system are known by means of the corresponding step responses, such as shown in Fig. 7. Thus, it is possible to predict their influences on the controlled outputs by means of step responses and the corresponding step response coefficients are incorporated in an augmented dynamic matrix. In this way, some predicted outputs based on the past disturbances can be calculated. A first attempt was performed by using the past disturbances exactly like the past manipulated inputs. Future measured disturbances could be considered as being constant and equal to the last measured disturbance in the same way as in Eq. (7). However, those attempts failed and a number of modifications have been introduced.

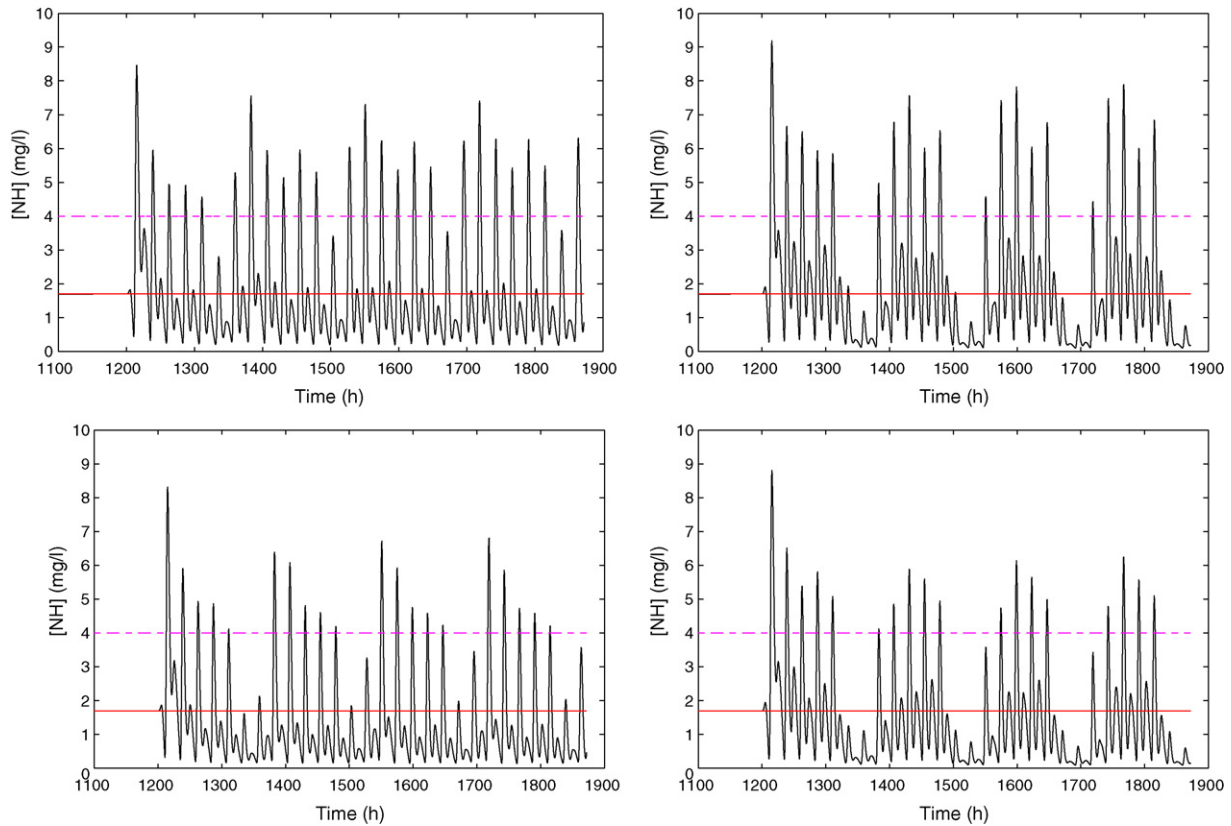
First, it is necessary to filter the disturbances in order to smooth their variations. A moving average filter with forgetting factor equal to 0.98 and of length  $H_m = 48$  (1 day) has been used. Other

forgetting factors equal to 0.95 and 0.9 have been tested without improvement. The length of the filter sequence was chosen in agreement with the model horizon and the apparent periodicity of the disturbances. The improvement of performance due to the influence of filtering is obvious in Table 4 where strategy DMC-FF- $q_0$ -1 corresponds to DMC without filtering and DMC-FF- $q_0$ -2 with filtering. All other control parameters were exactly the same. The same behavior was observed when QDMC is used without filtering or with filtering and only the results of strategy with filtering are given in QDMC-FF- $q_0$ . The impact of filtering on the effluent ammonium concentration is extreme and accompanied by a large increase of the aeration energy in the absence of filtering.

Secondly, the equations of the predictors have been modified with regard to the terms taking into account the measured disturbances which are not considered exactly in the same as the manipulated inputs. In these conditions, Eq. (2) of the output prediction  $\hat{y}(k+l|k)$  is modified as the sum of a steady-state term, a term as effect of past inputs, a term as effect of future inputs, a term as effect of past measured disturbances  $d_m$  and a term of unknown disturbances  $d_u$ :

$$\begin{aligned} \hat{y}(k+l|k) = & y_{ss} + \sum_{i=l+1}^{H_m-1} h_i \Delta u(k+l-i) + h_{H_m}(u(k+l-H_m) - u_{ss}) \\ & + \sum_{i=1}^l h_i \Delta u(k+l-i) + \sum_{i=l}^{H_{ff}-1} h'_i \Delta d_m(k+l-i) \\ & + \hat{d}_u(k+l|k) \end{aligned} \quad (23)$$

where  $H_{ff}$  is the feedforward horizon, not necessarily equal to the model horizon  $H_m$ . In Eq. (23), for the feedforward action, the role of



**Fig. 10.** Comparison of the influence of feedback and feedforward DMC strategies on the effluent ammonium concentration. Top, left: simple feedback. Top, right: feedforward on  $q_0$ . Bottom, left: feedforward on  $[\text{NH}]_0$ . Bottom, right: feedforward on  $q_0$  and  $[\text{NH}]_0$ . The continuous line is the set point. The dashed line is the upper bound.

the measured disturbances  $d_m$  is similar to that of the past inputs  $\Delta u$ , whereas the term  $d_u$  stands only for the unmeasured disturbances or unmodelled disturbances. The future disturbances will be part of the future unmeasured disturbances estimated as

$$\hat{d}_u(k+l|k) = \hat{d}_u(k|k) = y(k) - \hat{y}^*(k|k) \quad (24)$$

with the output prediction based on past inputs:

$$\hat{y}^*(k|k) = y_{ss} + \sum_{i=1}^{H_m-1} h_i \Delta u(k-i) + h'_{H_m} (d_m(k) - d_{m,ss}) \quad (25)$$

incorporating the present measured disturbance  $d_m(k)$  and its steady-state value  $d_{m,ss}$ .

Thus, the output prediction  $y^*(k+l|k)$  based on past inputs and past measured disturbances is defined as

$$y^*(k+l|k) = y_{ss} + \sum_{i=l+1}^{H_m-1} h_i \Delta u(k+l-i) + h_{H_m} (u(k+l-H_m) - u_{ss}) + \sum_{i=l+1}^{H_{ff}-1} h'_i \Delta d_m(k+l-i) \quad (26)$$

Now, in a similar way to Eq. (8), the future errors can be defined as

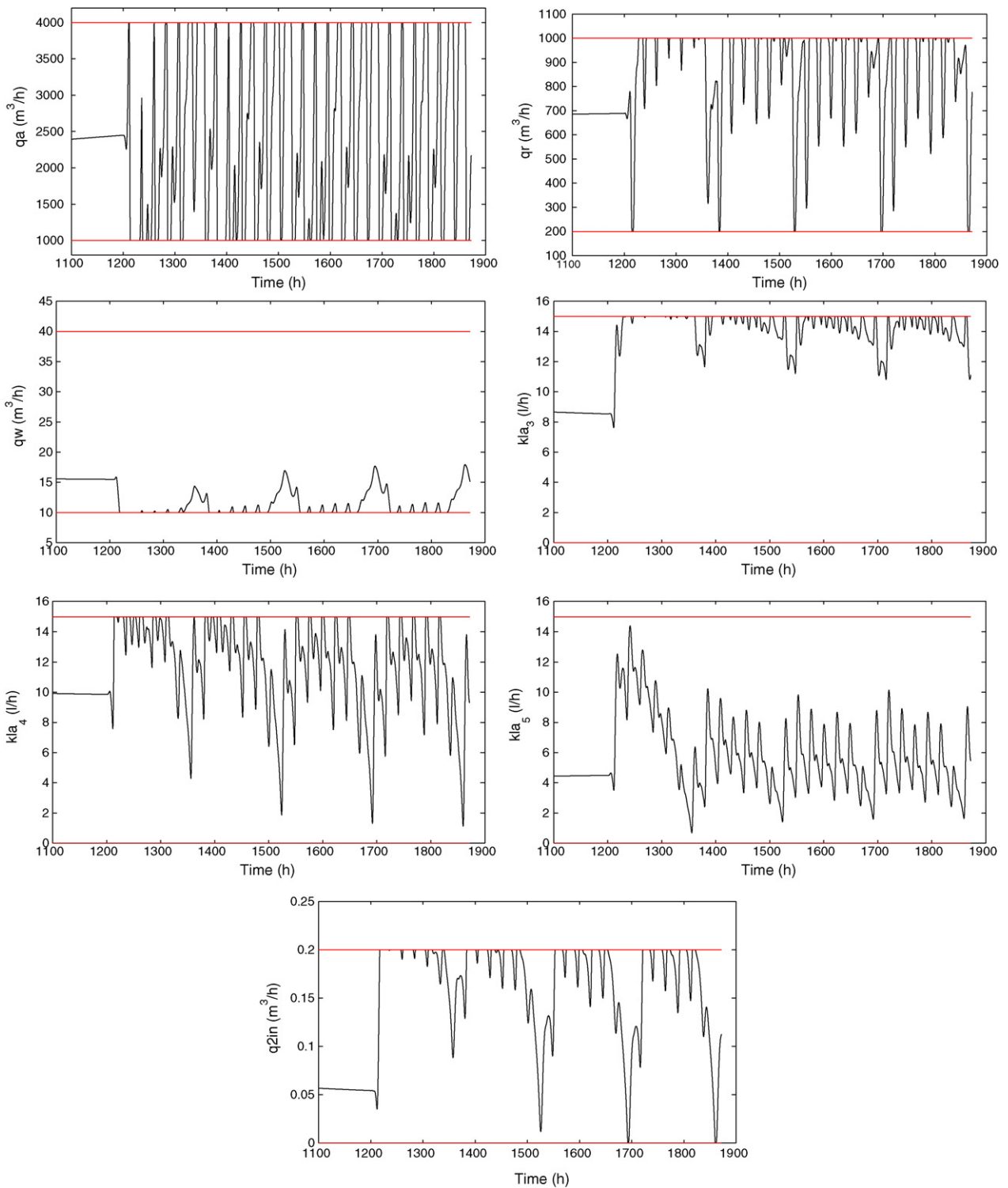
$$e(k+l) = y^{\text{ref}}(k+l) - y^*(k+l|k) - \hat{d}_u(k|k) \quad (27)$$

Globally, Eq. (4) is modified as

$$\begin{bmatrix} \hat{y}(k+1|k) \\ \vdots \\ \hat{y}(k+H_p|k) \end{bmatrix} = \begin{bmatrix} y^*(k+1|k) \\ \vdots \\ y^*(k+H_p|k) \end{bmatrix} + \mathcal{A} \begin{bmatrix} \Delta u(k) \\ \vdots \\ \Delta u(k+H_c-1) \end{bmatrix} + \begin{bmatrix} \hat{d}_u(k+1|k) \\ \vdots \\ \hat{d}_u(k+H_p|k) \end{bmatrix} \quad (28)$$

where a separation of measured and unmeasured disturbances has been introduced.

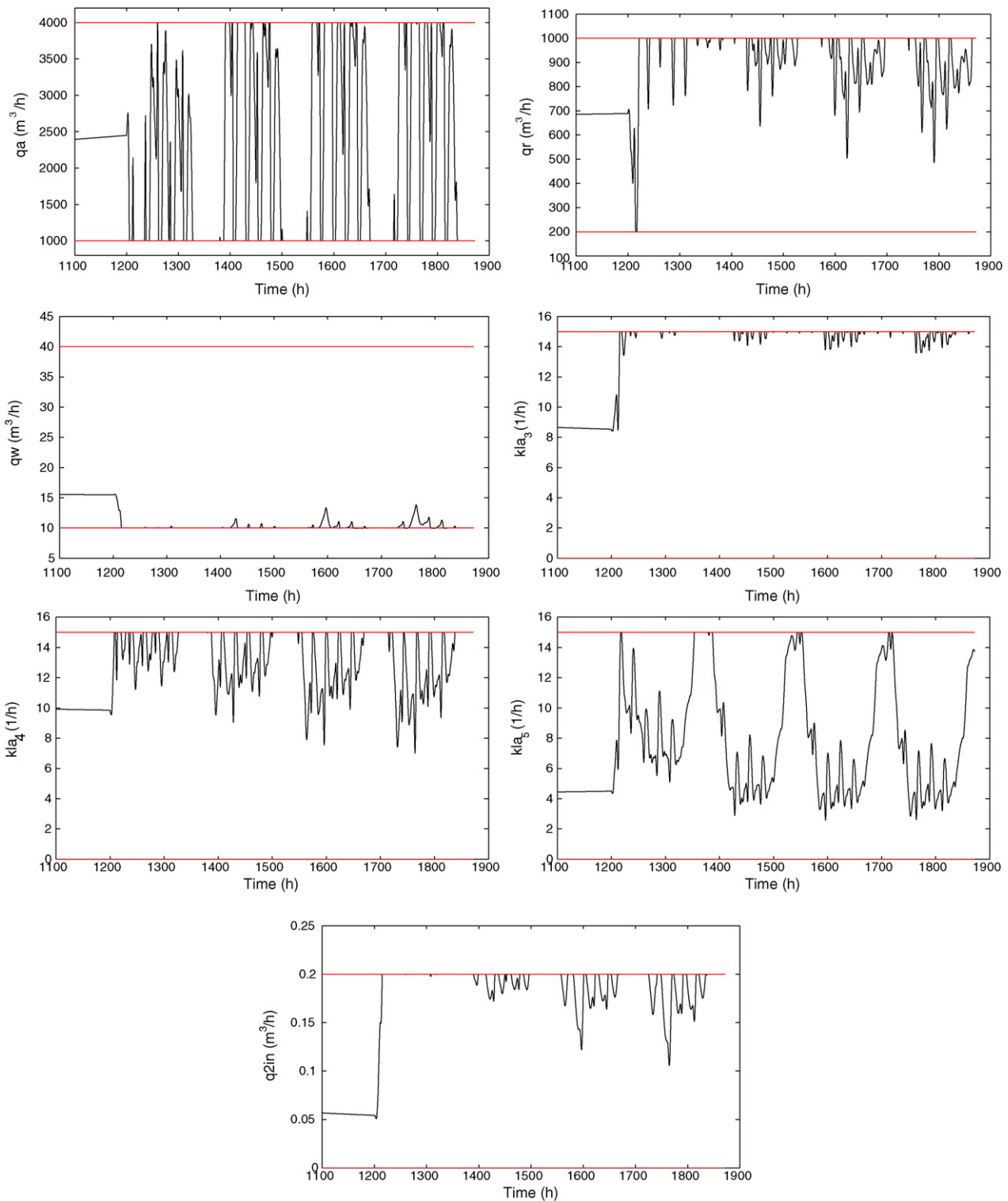
Recall that the feedforward control is introduced to counteract the influence of either the influent flow rate  $q_0$ , or the influent ammonium concentration  $[\text{NH}]_0$  or even both of them considered together. The feedforward strategies have been systematically tested with feedback DMC for different values of the feedforward horizon. The results are shown in Fig. 9 and quantitative results are given in Table 4. As the favorable influence of disturbance filtering by moving average filter with forgetting factor has already been demonstrated, all following cases are performed with disturbance filtering. For values of the feedforward horizon  $H_{ff}$  lower than 20, the performances of the different controllers are little influenced. When  $H_{ff}$  exceeds 20, in general, a marked degradation occurs (Fig. 9). The worst performance concerning the effluent ammonium concentration is reached by DMC with simple feedback (strategy DMC-FB-1), slightly improved by DMC with feedforward on  $q_0$  (strategy DMC-FF- $q_0$ -2), neatly improved by DMC with feedforward on  $[\text{NH}]_0$  (strategy DMC-FF- $[\text{NH}]_0$ -2) and again slightly improved by DMC with feedforward on both  $q_0$  and  $[\text{NH}]_0$  (strategy



**Fig. 11.** Manipulated inputs in the case of simple DMC without feedforward. From top to bottom, and from left to right: (a) internal recycle flow rate, (b) external recycle flow rate, (c) wastage flow rate, (d) oxygen mass transfer coefficient in the 3rd aerated unit, (e) oxygen mass transfer coefficient in the 4th aerated unit, (f) oxygen mass transfer coefficient in the 5th aerated unit, and (g) carbon source supplementation flow rate.

DMC-FF- $q_0$ -[NH]<sub>0</sub>). Thus, by far, the most important disturbance to be compensated is the influent ammonium concentration [NH]<sub>0</sub>, which is also visible on the open loop response (Fig. 7). The improvement due to the consideration of the influent  $q_0$  in a feedforward controller is noticeable, but less important. It must also be noticed that the improvement of the performance with respect to the effluent ammonium concentration is performed at the expense of a marked increase of the aeration energy as expected. The respective

influences of the different strategies on the effluent ammonium concentration are gathered in the single Fig. 10. The improvement of the profile of the effluent ammonium concentration is very noticeable when the feedforward strategy on both  $q_0$  and [NH]<sub>0</sub> is used. Because of the weights on the outputs that were chosen in the criterion (Table 1), the feedforward effect is much more visible on the decrease of ammonium and nitrogen concentration which were considered as being of major importance. The



**Fig. 12.** Manipulated inputs in the case of DMC with feedforward with respect to the influent flow rate and ammonium concentration. From top to bottom, and from left to right: (a) internal recycle flow rate, (b) external recycle flow rate, (c) wastage flow rate, (d) oxygen mass transfer coefficient in the 3rd aerated unit, (e) oxygen mass transfer coefficient in the 4th aerated unit, (f) oxygen mass transfer coefficient in the 5th aerated unit, and (g) carbon source supplementation flow rate.

solid constant lines in Fig. 10 correspond to the set points that should be followed because of environmental norms. Even with the best feedforward controller (strategy DMC-FF- $q_0$ -[NH] $_0$ ), the upper bounds represented by dashed lines in Fig. 10 are sometimes violated, however less often than with the feedback DMC (strategy DMC-FB-1).

The influent ammonium concentration is so important that the manipulated inputs move considerably after  $t = 1200$  h, when disturbances are present. Two sets of figures are presented to emphasize the difference between simple feedback DMC (strategy DMC-FB-1) (Fig. 11) and DMC with feedforward on both  $q_0$  and [NH] $_0$  (strategy DMC-FF- $q_0$ -[NH] $_0$ ) (Fig. 12). For all inputs,

the saturation limits are reached much more often with strategy DMC-FF- $q_0$ -[NH]<sub>0</sub> than with strategy DMC-FB-1, in spite of the large domain which was attributed for each manipulated input. The stronger variations of  $kl a_5$  are particularly noticeable. Thus, to be able to decrease the mean ammonium and total nitrogen concentrations, the feedforward strategy demands much energy. This is also confirmed in Table 4 where the aeration energy of feedforward strategies DMC-FF- $q_0$ -2, DMC-FF-[NH]<sub>0</sub>-2 and DMC-FF- $q_0$ -[NH]<sub>0</sub> respectively exceed by 7%, 9% and 16% the aeration energy of feedback control DMC-FB-1.

## 7. Conclusions and future work

This paper outlines the results of MPC strategy using BSM1 simulation benchmark of wastewater treatment plant. The strategies of linear DMC, QDMC and NLMPC in the case of without and with feedforward compensation have been tested. After a period of steady influent characteristics, large disturbances of the influent characteristics taken from a dry weather file are imposed. The simulation results presented in this paper indicate that all the model predictive controllers perform well during the first period of steady influent. However, in the presence of large disturbances, the control performance is extremely modified. Even, feedback DMC needs to impose bounds on the inputs at control time to avoid explosion. QDMC does not bring any advantage compared to DMC. NLMPC with strong penalty on the effluent ammonium and total nitrogen improves the performance with respect to these latter at the expense of more energy. Finally, the modified feedback DMC proves to be very satisfactory even when compared to the more complex QDMC and even to NLMPC. Feedforward control with respect to the measurable disturbances of the influent ammonium concentration or flow rate or both of them has been added to feedback DMC under a form close to that of the past manipulated inputs in the expression of the predicted outputs. It has been found that the disturbances need to be filtered, which is performed by a moving average filter with forgetting factor. Moreover, a specific horizon different from the model horizon had to be included for the use of the measured disturbances in the feedforward controller. As expected from the open loop responses, feedforward with respect to ammonium concentration is much more efficient than feedforward with respect to flow rate. However, this is performed at the expense of a larger aeration energy consumption. The best feedforward strategy is obtained by consideration of both influent ammonium concentration and flow rate as measured disturbances in the feedforward strategy. Also, in the feedforward case, the bounds on the inputs are very often reached.

The influent flow rate can be easily measured on a plant and is a good indication of changes in the wastewater characteristics related to human activity. However, the influent ammonium concentration has more influence than the influent flow rate, so that in the case where the influent ammonium concentration is not measured, its estimation might require the development of an observer, based on several other easier measurements.

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